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To What Extent Defining a Group Predicates on Defining Other Groups?

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Abstract

We present a framework of group cooperation and competition in which agents are concerned not only about their material payoffs but also about their psychological payoffs, derived from working with others *per se*. In such a framework, a group's psychological preferences serve to enhance the group's material payoffs. We show that a small group has strong incentives to engage in outward-looking identity strengthening, such as stereotyping or airing grievances against a specific, large outgroup, and a large group has strong incentives to engage in inward-looking identity strengthening, such as self-stereotyping, glorifying its own history, etc..

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1. Introduction

Intergroup relationship has been a core area in social psychology and a vast literature has evolved to address questions pertinent to it. Specifically, a question frequently raised is: To what extent defining a group predicates on defining other groups. According to Brewer & Miller (1996):

“Discussion of the importance of meaningful intergroup differentiation as a determinant of social identification raises the issue of the social context within which ingroups are defined... ‘[w]e are what we are because they are not what we are...’ But who constitutes this ambiguous ‘they’? Are relevant others limited to members of specific contrasting groups, or could ‘they’ refer to all other human beings who are excluded from membership in the ingroup?... Both theory and research are ambiguous on this issue of the need for specific outgroups as a factor in ingroup identity.” (pp. 47-48)

In this paper we present a group-theoretic model to shed light on this question. Imagine a community populated by agents with different characteristics (both economic and non-economic). They are partitioned into exhaustive and

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mutually exclusive groups so that those in the same group share the same characteristics (including preferences). Entry into and exit out of a group is forbidden so that no asymmetry between different dimensions of identity (class, race, religion, etc.) is presupposed. Agents interact among themselves to generate material production, obtaining material payoffs; they also derive (positive or negative) utility from the interactions *per se*, obtaining what we call psychological payoffs. Because agents in the same group are of the same characteristics, they may, for instance, enjoy greater psychological payoffs when working with each other than with any outgroup member. We use the term *group identity*¹ to describe the psychological preferences shared in a group.

The key ingredient of the model is that material payoffs and psychological payoffs are substitutes. Whereas the concern about the former compels agents to work together, the concern about the latter strengthens their bargaining power in their share of output. Hence, psychological preferences play a subtle role in affecting a group's material payoffs.² In this paper, we study a group's incentive to strengthen its own identity, through means such as education, propaganda, etc.. Two strategies are compared ---- one that increases the group's amicability among members and the other that decreases its amicability towards some judiciously chosen out-group. The former is meant to be one that does not require specific knowledge about outgroup members, while the second is meant to be one that requires such knowledge of a specific outgroup. We find that a large group benefits more from the former and a small group more from the latter. This thus sheds light on the question to what extent defining a group predicates on defining other groups.

A companion paper (Chiu & Zhong, 2011) further clarifies the material foundation to the psychological preferences introduced in this framework. It also applies the framework to study two contemporary phenomena of interest to social psychologists. The first is the declining importance of class in contemporary politics in developed economies. A widely accepted explanation is that class conflict is greatly attenuated because voters are concerned also about non-economic issues like religion, race, abortion, etc.. What remains to be explained is how the multi-dimensionality of preferences is determined, and to what extent it is "manufactured" by the rich. The second phenomenon is the salience of racial/ethnic conflict over class conflict in third-world countries. The pork theory, as proposed by Fearon (1999) and Caselli & Coleman II (2010), starts with the observation that unlike other social dimensions race is the easiest to be recognized while the hardest to be changed. Thus race-based coalitions provide the strongest warranty for agents to share the "pork" *ex post* and hence the strongest incentive for them to grip it *ex ante*. Chiu & Zhong (2011) generate interesting implications consistent with these two phenomena without using the existing economic theories.

2. Model

Our model follows the model in Chiu & Zhong (2011) closely. Consider a community consisting of a set of agents of measure N partitioned into n exhaustive and mutually exclusive groups; without confusion, we use $1, \dots, n$ to denote these groups. We use s_1, \dots, s_n to denote the measure of membership in each group. We assume that members in the same group have already overcome their collection action problem. As a result, we can adopt the following assumption.

Assumption 1: *Each group acts as a single decision maker.*

Agents produce output according to a characteristic function $v(\cdot)$: the value of the output produced by group i when it works alone is $v(i)$, the value of output produced jointly by group i and group j is $v(i \cup j)$, etc.. The characteristic function satisfies the following standard property (see, for instance, Shapley, 1953).

¹ It can be alternatively called group solidarity or group loyalty.

² Other major theories in intergroup bias also have the feature that an agent's psychology is useful for or enhances the agent's survival. Hewstone et al. (2002) succinctly summarize the following five major theories: social identity (Tajfel & Turner, 1979), optimal distinctiveness (Brewer, 1991), subjective uncertainty reduction (Hogg, 2000; Hogg & Abrams, 1993), terror management (Solomon et al., 1991), and social dominance (Sidanius & Pratto, 1999).

Assumption 2: The characteristic function $v(\cdot)$ is strictly superadditive, i.e., $v(R_1 \cup R_2) > v(R_1) + v(R_2)$, where R_1 and R_2 are any two disjoint collections of groups.

Strict superadditivity corresponds to the scenario where agents are strictly complementary in production and, as a result, formation of and cooperation in the grand coalition is socially optimal. In addition to material products, agents are also concerned about with whom they work; more specifically, agents derive psychological utility from working together *per se*. An interesting special case is homophily, i.e., agents prefer working with other agents in the same group. In the context of ethnicity, for instance, an individual may feel more comfortable dealing with a member of her ethnic group than with a non-member; in the context of religion, an individual may feel more comfortable dealing with someone who shares the same religious belief than someone who does not.

More generally, we represent group i 's psychological preferences by a vector $\mathbf{a}_i \equiv (a_{i1}, \dots, a_{in})$ of coefficients characterizing the group's intragroup and intergroup amicability. Put differently, a_{ij} measures how much group i members want to work with group j members. By forming a coalition R with some other groups, group i will obtain psychological payoffs

$$\alpha_i(R) \equiv s_i \times \left(\frac{s_i}{s_R} \times a_{ii} + \sum_{j: j \subset R \setminus i} \frac{s_j}{s_R} \times a_{ij} \right), \tag{1}$$

where $s_R = \sum_{j \subset R} s_j$.

The right-hand side (RHS) of (1) has a natural interpretation. Within a certain period of time, members in the coalition engage in pairwise matching so that each member spends an equal amount of time with every other member. For a member in group i , in particular, she will spend s_i / s_R of her time with group i 's members, and s_j / s_R of her time with group j 's members, where $j \subset R \setminus i$. This accounts for the term in the parentheses on the RHS of (1). Given s_i members in group i , the RHS represents the total psychological payoffs that group i members will collectively obtain when coalition R is formed. If $R = i$, i.e., if group i chooses to work alone, its psychological payoffs will simply be $\alpha_i(i) = s_i a_{ii}$.

The way we model the utility function follows that of Alesina & La Ferrara (2000), who study residents' decisions to contribute to a public good in a racially heterogeneous community. In their model, each resident's utility is assumed to depend on the proportion of residents of the same race as himself or herself in the whole population. Using US survey data on attitude toward redistribution, Luttmer (2001) finds that, controlling for income, individuals increase their support for welfare spending as the share of local recipients from their own racial group rises. Luttmer refers to this as group loyalty. We think that assuming some kind of psychological payoffs when race is concerned is a good short-cut in the modeling.

We call the n -vector \mathbf{a}_i as group i 's *identity* and the components of the vector as the group's *identity coefficients* or *amicability coefficients*. Given $\mathbf{a}'_i \equiv (a'_{i1}, \dots, a'_{in})$ and $\mathbf{a}''_i \equiv (a''_{i1}, \dots, a''_{in})$ if $a'_{ii} \geq a''_{ii}$ and $a'_{ij} \leq a''_{ij}$ for any $j \neq i$, we say that group i has a stronger identity under \mathbf{a}'_i than under \mathbf{a}''_i , or \mathbf{a}'_i is an (*identity*) *strengthening* of \mathbf{a}''_i . (Note that $\mathbf{a}_i = (a_{i1}, \dots, a_{ii}, \dots, a_{in})$ with $a_{ii} > a_{ij}, \forall j \neq i$ corresponds to the aforementioned phenomenon of homophily.) Each group's identity coefficients are exogenously given, and our main

exercise is to perform comparative statics of group welfare with respect to these coefficients.³

Next, we assume that a group’s total payoffs equal the sum of its material payoffs and psychological payoffs. Thus, given a coalition R , the total utility of its member groups is given by

$$u(R) \equiv v(R) + \sum_{j: j \subset R} \alpha_j(R).$$

We call $u(\cdot)$ the total characteristic function.

Assumption 3: *The total characteristic function $u(\cdot)$ is strictly superadditive, i.e., $u(R_1 \cup R_2) > u(R_1) + u(R_2)$, where R_1 and R_2 are any two disjoint unions of groups.*

Assumption 3 states that, because the absolute values of α_{lm} are moderate enough for all l and m , even when psychological payoffs are also taken into account, the formation of the grand coalition is still efficient. It is worth noticing that most research in social psychology about intergroup bias focuses on milder degree of bias (see the survey by Hewstone et al., 2002).

Given the total utility function, we assume that each group obtains its own Shapley value taking each group as an individual player.⁴ Assumption 1 justifies the treatment that each group enters the bargaining as a single decision maker. Assumption 3 implies that forming the grand coalition is indeed efficient.

More specifically, group i will obtain a total payoff of

$$\phi_i(N) \equiv \sum_{T: i \in T} \frac{|T|!(n-|T|-1)!}{n!} [u(T \cup i) - u(T)], \tag{2}$$

where $|T|$ is the number of groups in coalition T . There is a natural interpretation here. Imagine that groups arrive at the scene in a random order, then (2) is just group i ’s weighted average of its marginal contribution to each conceivable coalition that it joins.

We use $\beta_i(N)$ to denote group i ’s material payoffs, where

$$\beta_i(N) \equiv \phi_i(N) - \alpha_i(N).$$

When our discussion is restricted to a smaller union of groups, $R \subset N$, the total payoffs and material payoffs that group i obtains are denoted by $\phi_i(R)$ and $\beta_i(R)$, respectively, and are calculated in a similar manner. To economize the notation, we define $\beta_i \equiv \beta_i(N)$, $\phi_i \equiv \phi_i(N)$, and $\alpha_i \equiv \alpha_i(N)$.

In what follows, we are interested in the effect of identity coefficients on the material payoff of groups. To this end, we study how an infinitesimal change in the former influences the latter. The change may be made possible by

³ Despite similarity, the notion of group identity we use is different from the notion of identity as is well known in the literature. The latter is formally introduced to economics in the seminal paper by Akerlof & Kranton (2000) and there, defined from an agent’s perspective, identity means the agent’s sense of self.

⁴ For Shapley value and its noncooperative game theory foundation, see Dasgupta & Chiu (1998).

education, media, subtle priming, etc..⁵ One can of course go further to take into account the costs incurred to change identity coefficients. If we consider a sufficiently convex cost function, there will then be a small extent to which identity is optimally changed.

3. Results

3.1. A material foundation of psychology

In Chiu & Zhong (2011), we find that a group's material payoff (i) is increasing in its intragroup amicability, as well as the intergroup amicability towards it; (ii) is decreasing in the intragroup amicability within and intergroup amicability among other groups; and (iii) somewhat surprisingly, may be increasing in its amicability toward some outgroup. The basic idea is that, by working with outgroup members, group members will be diluting their own interactions ---- as well as the interactions of outgroup members ---- and their bargaining power will be strengthened or weakened dependent on the various group identities. While results (i) and (ii) are fairly intuitive, result (iii) suggests intriguing counter-intuitive spillovers between groups and is worth restating formally here (proof omitted).

Lemma 1: For $j \neq i$ and $s_i > 0$ and $s_j > 0$,

1. $\partial\beta_i / \partial a_{ij} > 0$ (< 0) if $(s_i + s_j) / N < \frac{2}{n(n-1)+2}$ ($> \frac{1}{2} \frac{n-2}{n-1}$); and
2. for $n = 3$, $\partial\beta_i / \partial a_{ij} > 0$ if and only if $(s_i + s_j) / N < 1/4$.

Presupposition has it that loving others always hurts as long as material payoff is concerned. Lemma 1 says that this presupposition is not true if the combined size of the ingroup and the outgroup being loved is sufficiently small.

3.2. Identity strengthening

The material foundation of identity discussed in the previous section suggests that an individual group does have the incentive to modify and in most cases strengthen its identity. Consider the following two ways of identity strengthening (through media, education, etc.)

- Group i is said to engage in *stereotyping (or outward identity strengthening)* if it chooses to decrease its amicability (or increase its hostility) toward one particular out-group j , i.e., to decrease a_{ij} for one $j \neq i$.
- Group i is said to engage in *self-stereotyping (or inward identity strengthening)* if it chooses to increase its intragroup amicability, i.e., to increase a_{ii} .

The self-stereotyping strategy is built by nurturing the belief among the ingroup members that their group is unique and special, without referring to specific features of any outgroup. The stereotyping strategy goes exactly the other way. It is the very peculiar feature of group j that makes members in group i think they themselves are different. Throughout the whole reasoning, the focus is outgroup j but not ingroup i . We now compare the profitability of these two alternative strategies, by examining one-unit changes in the respective identity coefficients.

The material benefit of the self-stereotyping strategy is $\partial\beta_i / \partial a_{ii}$ while the material benefit of the stereotyping strategy targeting group j is $-\partial\beta_i / \partial a_{ij}$. Hence, as far as material payoff is concerned, the stereotyping strategy

⁵ A large literature in social psychology has grown to establish the social cognitive perspective in intergroup bias. Much of the research shows that, under most minimal conditions, respondents show ingroup preferences. "These findings appear to suggest that a history of intense dislike and conflict is not necessary to produce ethnocentric behavior. The mere division into groups suffices." (Park & Judd, 2005, pp. 109) For a critique of this view, see Park & Judd (2005); for recent evidence, see Gutierrez & Unzueta (2010), Morrison & Chung (2011), and Wolpin et al. (2011).

is more profitable if

$$\frac{\partial \beta_i}{\partial a_{ii}} < \max_j \left(-\frac{\partial \beta_i}{\partial a_{ij}} \right), \quad (3)$$

where the **max** operator implies that the targeted outgroup is judiciously chosen. It is straightforward to obtain the following lemma (see Appendix for proofs to the lemma and the next proposition).

Lemma 2: *Under Assumptions 1-3, we have*

$$\frac{\partial \beta_i}{\partial a_{ii}} = \sum_{j:j \neq i} \left(-\frac{\partial \beta_i}{\partial a_{ij}} \right). \quad (4)$$

Lemma 2 means that, in terms of group i 's material payoffs, increasing a_{ii} by one unit while holding a_{ij} constant for all $j \neq i$ is the same as decreasing a_{ij} by one unit for all $j \neq i$ while holding a_{ii} constant. Comparing (3) with (4), one can easily verify that (3) holds only if there exists some $k \neq i$ such that $\partial \beta_i / \partial a_{ik}$ is positive. Put differently, a necessary condition for the outward strategy to be the optimal strategy for group i is that there exists some group k , $k \neq i$, such that group i can benefit from increasing its amicability towards it. The condition for this latter scenario was given in Lemma 1. Making use of it, we obtain the following proposition.

Proposition 1: *Under Assumptions 1-3, we have*

1. for $n \geq 4$, $\partial \beta_i / \partial a_{ii} > \max_j (-\partial \beta_i / \partial a_{ij})$ if (i) $s_i / N > \frac{1}{2} \frac{n-2}{n-1}$ or (ii) s_j is the same for all $j \neq i$;
2. for $n \geq 3$, $-\partial \beta_i / \partial a_{ij} < \partial \beta_i / \partial a_{ii}$ if $s_j / N < (2/n)(1 - s_i / N)$;
3. for $n \geq 3$, $-\partial \beta_i / \partial a_{ij} > \partial \beta_i / \partial a_{ii}$ if $s_j / N > \frac{n(n-1)}{n(n-1)+2}$;
4. for $n = 3$, $-\partial \beta_i / \partial a_{ij} > \partial \beta_i / \partial a_{ii}$ if and only if $s_j / N > 3/4$.

Result 1.i states that, assuming $n \geq 4$, the self-stereotyping strategy dominates when group i 's size is large enough (e.g. exceeding one third when $n = 4$) because the intragroup amicability now has an overwhelming weight in determining psychological payoffs. The same is true when all outgroups are symmetric in size so that no particular outgroup is important enough to be targeted towards (result 1.ii). Results 2 and 3, respectively, provide sufficient and necessary conditions, when $n \geq 3$, for the stereotyping strategy with targeted group j to dominate and to be dominated by the self-stereotyping strategy, respectively. When outgroup j is sufficiently small, the stereotyping strategy with j as the targeted group is dominated by the self-stereotyping strategy; when it is sufficiently large, the reverse is true. Result 4 gives the necessary and sufficient condition for the outward strategy to dominate the self-stereotyping strategy when there are only three groups i , j , and k . Because $n = 3$, there are only two terms in the RHS of (4), and the necessary condition is also the sufficient condition.

Despite a common presumption that a small group is usually the victim in an intergroup conflict, our result suggests that it need not be the case. It turns out that, in our framework, a small group has strong incentives to engage in outward-looking identity strengthening, such as stereotyping or airing grievances against a specific, large out-group, while a large group has strong incentives to engage in inward-looking identity strengthening, such as self-stereotyping and glorifying its own history. It suggests that when a large group seems to be bullying a small group, it may in fact only be a small segment of the former that is doing so. Glaeser (2005) models a small segment

of the agents, called political entrepreneurs, in the majority who spread hatred against a minority.

More generally, our analysis suggests that there is generally a difference between when well-defined outgroups exist and when they do not and clarifies the conditions under which knowledge of well-defined outgroups really matters. Having a well-defined outgroup is useful for a small ingroup because it allows the ingroup to have a target for stereotyping. On the other hand, such a well-defined outgroup is not necessarily for a large ingroup because the ingroup will find it more beneficial to engage in self-stereotyping.

4. Conclusion

In this paper, we have presented a framework of intergroup relationship in which agents are concerned about both material payoffs and psychological payoffs. The framework has the following feature. On the one hand, agents want to work with outgroup agents to increase their material payoffs. On the other hand, they are unwilling to do so unless they are sufficiently compensated for their losses. Hence, we are able to determine each group's welfare as a function of its own group identity, as well as the group identities of other groups. This thus allows us to study the incentive for a group to shape its own psychological preferences.

We have compared two strategies ---- one that increases the group's amicability among members (self-stereotyping) and the other that decreases its amicability towards some judiciously chosen out-group (stereotyping). Self-stereotyping does not require specific knowledge about outgroup members, while stereotyping requires such knowledge of a specific outgroup. We find that a large group benefits more from self-stereotyping and a small group more from stereotyping. Therefore, having a well-defined outgroup is useful for a small ingroup because it allows the ingroup to have a target for stereotyping; on the other hand, such a well-defined outgroup is not necessarily for a large ingroup because the ingroup will find it more beneficial to engage in self-stereotyping.

In our analysis, we have ignored the costs of changing a group's identity. Implicitly we assumed that the cost of strengthening intragroup amicability by one unit is just the same as the cost of decreasing intergroup amicability toward a targeted outgroup by one unit. Because of this, we could simply compare the effects of self-stereotyping and stereotyping without examining the costs. In reality, the aforementioned implicit assumption on costs does not hold in general. For instance, an ingroup may differ from outgroup 1 with a rigid boundary and from outgroup 2 with a fluid, superficial boundary. In this case, stereotyping against outgroup 1 may be easier, less costly, than stereotyping against outgroup 2. We leave these issues for future studies.

In the last two decades, economics has been learning a lot from psychology. In this paper, we have shown that, by using formal modeling that is familiar in economics but not in psychology, we can shed light on issues that interest psychologists. This is also part of the reason encouraging us to write this paper.

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Appendix A. Proof of Lemma 2

The RHS equals

$$-\sum_{j:j \neq i} \left[\sum_{T:i \in T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{s_i s_j}{s_T + s_i} \right) - \frac{s_i s_j}{N} \right]$$

$$\begin{aligned}
 &= - \sum_{T:i \notin T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i s_T}{s_T + s_i} + \frac{s_i(N-s_i)}{N} \\
 &= - \sum_{T:i \notin T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i(s_T + s_i - s_i)}{s_T + s_i} + s_i - \frac{s_i^2}{N} \\
 &= - \sum_{T:i \notin T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} s_i + \sum_{T:i \notin T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i^2}{s_T + s_i} + s_i - \frac{s_i^2}{N} \\
 &= - \frac{n! - (n-1)!}{n!} s_i + \sum_{T:i \notin T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i^2}{s_T + s_i} + s_i - \frac{s_i^2}{N} \\
 &= \frac{s_i}{n} + \sum_{T:i \notin T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i^2}{s_T + s_i} - \frac{s_i^2}{N} \\
 &= \sum_{T:i \notin T} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i^2}{s_T + s_i} - \frac{s_i^2}{N}
 \end{aligned}$$

which is just the LHS.

Appendix B. Proof of Proposition 1

Before showing Proposition 1, we first show two useful immediate results. Note that

$$\begin{aligned}
 \frac{\partial \phi_i}{\partial a_{ii}} &= \sum_{T:i \notin T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{\partial u(T \cup i)}{\partial a_{ii}} - \frac{\partial u(T)}{\partial a_{ii}} \right) \\
 &= \sum_{T:i \notin T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{\partial u(T \cup i)}{\partial a_{ii}} \right) \\
 &= \sum_{T:i \notin T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{s_i^2}{s_T + s_i} \right) > 0.
 \end{aligned}$$

Since $\partial \alpha_i / \partial a_{ii} = s_i^2 / N$, we then have

$$\begin{aligned}
 \frac{\partial \beta_i}{\partial a_{ii}} &= \frac{\partial \phi_i}{\partial a_{ii}} - \frac{\partial \alpha_i}{\partial a_{ii}} = \sum_{T:i \notin T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{s_i^2}{s_T + s_i} \right) - \frac{s_i^2}{N} \\
 &= \sum_{T:i \notin T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{s_i^2}{s_T + s_i} - \frac{s_i^2}{N} \right) > 0.
 \end{aligned} \tag{5}$$

Next, note that

$$\begin{aligned} \frac{\partial \phi_i}{\partial a_{ij}} &= \sum_{T:i \notin T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{\partial u(T \cup i)}{\partial a_{ij}} - \frac{\partial u(T)}{\partial a_{ij}} \right) \\ &\quad + \sum_{T:i, j \notin T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{\partial u(T \cup i)}{\partial a_{ij}} - \frac{\partial u(T)}{\partial a_{ij}} \right) \\ &= \sum_{T:i \notin T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{\partial u(T \cup i)}{\partial a_{ij}} \right) \\ &= \sum_{T:i \notin T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i s_j}{s_T + s_i} > 0. \end{aligned}$$

Since $\partial \alpha_i / \partial a_{ii} = s_i^2 / N$, we then have

$$\frac{\partial \beta_i}{\partial a_{ij}} = \frac{\partial \phi_i}{\partial a_{ij}} - \frac{\partial \alpha_i}{\partial a_{ij}} = \sum_{T:i \notin T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i s_j}{s_T + s_i} - \frac{s_i s_j}{N}. \tag{6}$$

For result 1.i, if $s_i / N > \frac{1}{2} \frac{n-2}{n-1}$, then $(s_i + s_k) / N > \frac{1}{2} \frac{n-2}{n-1}$ for all $k \neq i$. By Lemma 1.i, we have $\partial \beta_i / \partial a_{ik} < 0$. This fact and Lemma 2 together imply that, for any $j \neq i$,

$$-\frac{\partial \beta_i}{\partial a_{ij}} = \frac{\partial \beta_i}{\partial a_{ii}} + \sum_{k:k \neq i, j} \frac{\partial \beta_i}{\partial a_{ik}} < \frac{\partial \beta_i}{\partial a_{ii}}.$$

Therefore, $\max_j (-\partial \beta_i / \partial a_{ij}) < \partial \beta_i / \partial a_{ii}$. For result 1.ii, s_j being the same for all $j \neq i$, together with Lemma 2, implies that

$$-\frac{\partial \beta_i}{\partial a_{ij}} = \frac{1}{n-1} \frac{\partial \beta_i}{\partial a_{ii}} < \frac{\partial \beta_i}{\partial a_{ii}}$$

for all $j \neq i$. As such, we have

$$\max_j \left(-\frac{\partial \beta_i}{\partial a_{ij}} \right) = \frac{1}{n-1} \frac{\partial \beta_i}{\partial a_{ii}} < \frac{\partial \beta_i}{\partial a_{ii}}.$$

For result 2, by definition (from (6)),

$$\begin{aligned} -\frac{\partial \beta_i}{\partial a_{ij}} &= \frac{s_i s_j}{N} - \sum_{T:i \notin T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i s_j}{s_T + s_i} \\ &< \frac{s_i s_j}{N} - \sum_{T:i \notin T, j \in T} \frac{|T|!(n-|T|-1)!}{n!} \frac{s_i s_j}{N} = \frac{1}{2} \frac{s_i s_j}{N}. \end{aligned}$$

Also by definition (from (5)), we have

$$\begin{aligned} \frac{\partial \beta_i}{\partial a_{ii}} &= \frac{s_i}{n} + \sum_{T:i \in T, T \neq \emptyset} \frac{|T|!(n-|T|-1)!}{n!} \left(\frac{s_i^2}{s_T + s_i} \right) - \frac{s_i^2}{N} \\ &> \frac{s_i}{n} + \left(1 - \frac{1}{n} \right) \frac{s_i^2}{N} - \frac{s_i^2}{N} = \frac{s_i}{n} \left(1 - \frac{s_i}{N} \right). \end{aligned}$$

Therefore, a sufficient condition for $-\partial \beta_i / \partial a_{ij} < \partial \beta_i / \partial a_{ii}$ is to let

$$\frac{s_i}{n} \left(1 - \frac{s_i}{N} \right) > \frac{1}{2} \frac{s_i s_j}{N} \Leftrightarrow \frac{s_j}{N} < \frac{2}{n} \left(1 - \frac{s_i}{N} \right).$$

For result 3, if $s_j / N > \frac{n(n-1)}{n(n-1)+2}$, then $(s_i + s_k) / N < \frac{2}{n(n-1)+2}$ for any $k \neq i, j$. By Lemma 1.i again, this implies $\partial \beta_i / \partial a_{ik} > 0$. Using this fact and Lemma 2, we get

$$-\frac{\partial \beta_i}{\partial a_{ij}} = \frac{\partial \beta_i}{\partial a_{ii}} + \sum_{k:k \neq i, j} \frac{\partial \beta_i}{\partial a_{ik}} > \frac{\partial \beta_i}{\partial a_{ii}}.$$

For result 4, we can derive directly from (6) and (5) the expression of $-\partial \beta_i / \partial a_{ij}$ and $\partial \beta_i / \partial a_{ii}$ in the three-group case as follows:

$$\begin{aligned} -\frac{\partial \beta_i}{\partial a_{ij}} &= \frac{s_i s_j}{N} - \frac{1}{6} \frac{s_i s_j}{s_i + s_j} - \frac{1}{3} \frac{s_i s_j}{s_i + s_j + s_k} = \frac{2}{3} \frac{s_i s_j}{N} - \frac{1}{6} \frac{s_i s_j}{s_i + s_j} \\ \frac{\partial \beta_i}{\partial a_{ii}} &= \frac{s_i}{3} + \frac{1}{6} \frac{s_i^2}{s_i + s_j} + \frac{1}{6} \frac{s_i^2}{s_i + s_k} + \frac{1}{3} \frac{s_i^2}{s_i + s_j + s_k} - \frac{s_i^2}{N} \\ &= \frac{s_i}{3} + \frac{1}{6} \frac{s_i^2}{s_i + s_j} + \frac{1}{6} \frac{s_i^2}{s_i + s_k} - \frac{2}{3} \frac{s_i^2}{N}. \end{aligned}$$

Therefore, the necessary and sufficient condition for $-\partial \beta_i / \partial a_{ij} > \partial \beta_i / \partial a_{ii}$ can be obtained as:

$$\begin{aligned} \frac{2}{3} \frac{s_i s_j}{N} - \frac{1}{6} \frac{s_i s_j}{s_i + s_j} &> \frac{s_i}{3} + \frac{1}{6} \frac{s_i^2}{s_i + s_j} + \frac{1}{6} \frac{s_i^2}{s_i + s_k} - \frac{2}{3} \frac{s_i^2}{N} \\ \Leftrightarrow 4 \frac{s_j}{N} - \frac{s_j}{s_i + s_j} &> 2 + \frac{s_i}{s_i + s_j} + \frac{s_i}{s_i + s_k} - 4 \frac{s_i}{N} \text{ (canceling } s_i/6) \\ \Leftrightarrow 4 \frac{s_i}{N} + 4 \frac{s_j}{N} - \frac{s_i / N}{1 - s_j / N} - 3 &> 0 \end{aligned}$$

$$\Leftrightarrow \left[\frac{s_j}{N} - \left(1 - \frac{s_i}{N} \right) \right] \times \left[\frac{s_j}{N} - \frac{3}{4} \right] < 0$$

$$\Leftrightarrow \frac{3}{4} < \frac{s_j}{N} < 1 - \frac{s_i}{N},$$

the upper bound of which is fulfilled by construction. This concludes the proof.

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